## Supplementary exercise 7.4 of IPS7e

An unspecified dataset with $n=30$ observations (presumably assumed to be i.i.d. and to be normally distributed) and a $t$-test value of $t_{\text{obs}}=1.12$ for

$$\text{H}_0 : \mu = 64 \quad \text{against the alternative hypothesis} \quad \text{H}_a : \mu \neq 64\,.$$

(a) The degrees of freedom equals: $\text{df} = n-1 = 29$.

(b) In Table C of PSLS, the value 1.12 falls in-between the values $t_1 = 1.055$ and $t_2 = 1.311$ in the row for 29 degrees of freedom, or more formally: $t_1 = 1.055 < t_{\text{obs}} < 1.311 = t_2$. The values are termed "critical values" in the table, but this terminology relates to statistical testing at different significance levels and is not necessarily helpful. In the terminology of the question, these values are also labelled as $t^*$, but that is probably more useful in the context of confidence intervals (where $t^*$ values are used).

The right-tail (or upper-tail) probabilities for $t_1$ and $t_2$ are 0.15 and 0.10, respectively, meaning that $t_1$ and $t_2$ are the 85% and 90% percentiles of the $t(29)$-distribution. The upper-tail probabilities can be found in the bottom of each column in the table, under "One-Sided P". The bounds for $t_{\text{obs}}$ imply that

$$0.10 < \text{P}(t(29) > t_{\text{obs}}) < 0.15.$$

(c) Because of the two-sided alternative hypothesis, the $P$-value equals twice the upper-tail probability, so we get

$$0.20 = 2 \cdot 0.10 < 2 \cdot \text{P}(t(29) > t_{\text{obs}}) < 2 \cdot 0.15 = 0.30.$$

These bounds are conveniently found at the bottom of the table under "Two-Sided P". The $P$-value could be reported as this, or one could limit the reporting to the relevant one for assessing significance at the most common significance levels, including 0.05, namely that: $P > 0.20$.

(d) We saw above that $P > 0.20$. Hence the test is not significant at the 10% level and nor at the 5% level (in fact, not being significant at the 10% level, means that it's not significant at any lower significance levels).

(e) In Minitab, the most conveninent menu is Graph-Probability Distribution Plot. Select View Probability and the $t(29)$ distribution, and under shared area insert as "X Value" the value 1.12 and pick either Right Tail or Both Tails to see that the right-tail probability equals 0.1360. Hence, the $P$-value equals $P = 2 \cdot 0.1360 = 0.272$.

