

Mid-term exam, 23 October 2019

All aids are allowed, except a computer-like device (including tablets and smartphones) and personal assistance. The exam consists of one question with five parts, or subquestions, labeled by letters **a)-e)**, that should all be answered to achieve the maximal 15% score. The different parts have equal weights (as indicated), and can be answered independently of each other and in any order. The mid-term exam accounts for 15% of the course mark; however, every student may choose to waive the result of the mid-term exam. The duration of the mid-term exam is 1 hour.

Generally, **statistical models and methods should be specified**, and data should be described graphically whenever that would be useful/relevant. If you realize that you need more information than is provided to carry out the analysis, specify what information you need, how you would obtain it using Minitab/Stata, and how you would use it.

Question 1

Following an incident during a university football game in 1993 where one of the players produced an unbelievably long shot (kick), it was investigated whether the football had some defect; in particular, the claim was made that the football had been filled with helium instead of air, so as to explain the extreme shot. Although the claim was later disproved, interest arose in whether helium-filled footballs really would travel further than air-filled footballs. We consider here results from two separate studies that were carried out to explore the issue.

In *Study 1*, two footballs — one filled with helium and one filled with air — were kicked eight times each: four times with the wind and four times against the wind. The kicker was aware which of the two footballs he was kicking.

The reported findings of the study were: the helium-filled football travelled on the average 10 yards¹ farther with the wind, while against the wind it travelled an average of 5 yards less.

In *Study 2*, two footballs — one filled with helium and one filled with air, but identical in appearance — were used outdoors on a windless day. Each football was kicked 39 times by a novice player, who was unaware of the identity of the two footballs, so that the two footballs were alternated with each kick.

Summary findings of the study were: mean and median distances of 26 yards for the air-filled football, and mean and median distances of 26.4 and 28 yards for the helium-filled football.

- a)** (3 points) Describe and contrast, possibly in bullet list form, the design of the two studies. Make sure to address the study type (e.g. observational) and the population the observations in each study would seem to represent. Discuss critically the advantages and disadvantages of the two studies relative to each other. Based on the information provided, does Study 2 in your view refute the findings of Study 1? Explain your reasoning (briefly).

¹ 1 yard equals approximately 0.914 m.

The remaining questions are all about the results of Study 2. For reference, a full listing of the data is shown below. The variables contain the distances (measured in yards) of each of the 39 kicks of the two footballs, filled either with air or helium. The listing also includes the difference, computed as: “Helium” minus “Air”.

Trial	Air-filled	Helium-filled	Difference	Trial	Air-filled	Helium-filled	Difference
1	25	25	0	21	31	31	0
2	23	16	-7	22	27	34	7
3	18	25	7	23	22	39	17
4	16	14	-2	24	29	32	3
5	35	23	-12	25	28	14	-14
6	15	29	14	26	29	28	-1
7	26	25	-1	27	22	30	8
8	24	26	2	28	31	27	-4
9	24	22	-2	29	25	33	8
10	28	26	-2	30	20	11	-9
11	25	12	-13	31	27	26	-1
12	19	28	9	32	26	32	6
13	27	28	1	33	28	30	2
14	25	31	6	34	32	29	-3
15	34	22	-12	35	28	30	2
16	26	29	3	36	25	29	4
17	20	23	3	37	31	29	-2
18	22	26	4	38	28	30	2
19	33	35	2	39	28	26	-2
20	29	24	-5				

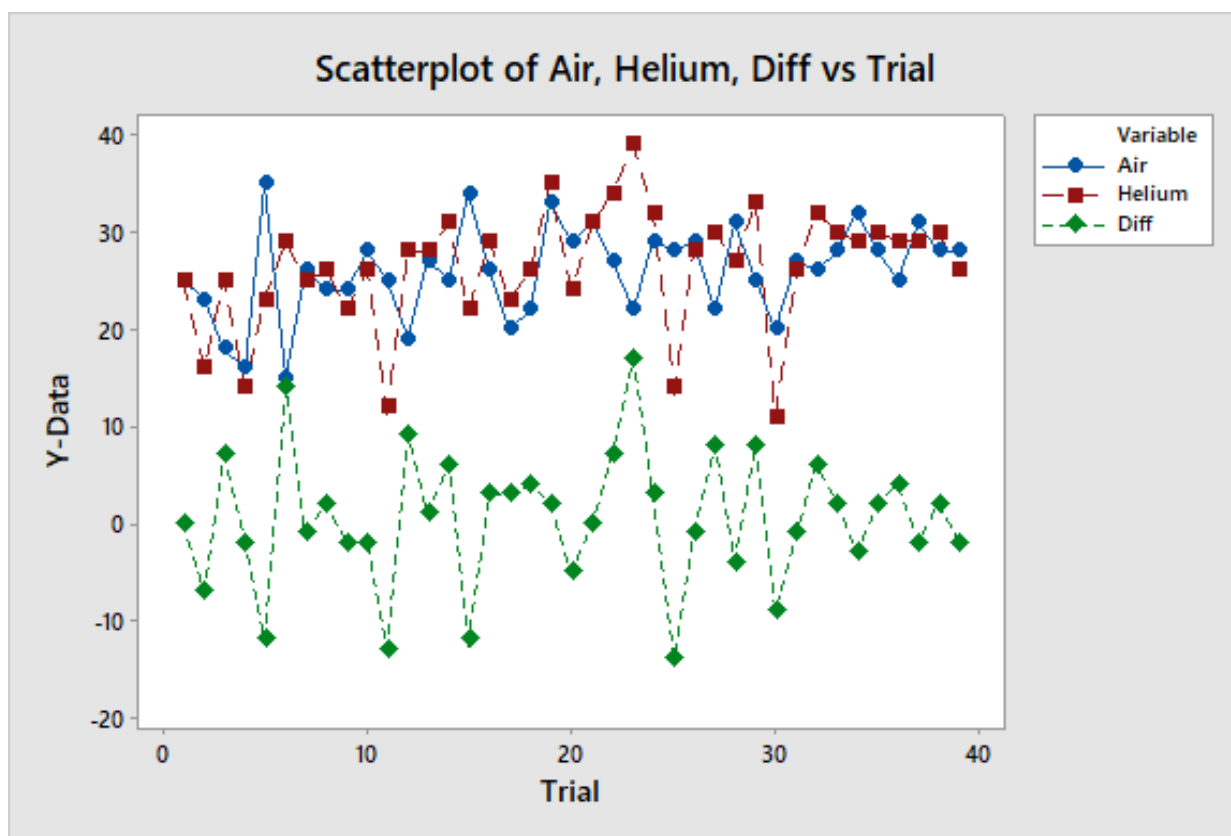
Use the information above as well as the subsequent software listings for parts **b)** – **e)**.

- b)** (3 points) In a newspaper article about Study 2, it was stated that “the air-filled ball went an average of 26.0 yards (plus or minus 0.7 yards); the helium-filled ball 26.4 yards (plus or minus 1 yard)”. First, without looking at the data, try to interpret the meaning of the statement in statistical terms (specifically, is the plus/minus range for the individual values or for the mean?) Second, try to reconstruct how the journalist arrived at this statement (that is, how were the reported values computed and what do they represent?). Third, try to modify the statement (for one of the two footballs) by rewording and/or altering the values given, so that its correct statistical interpretation becomes more clear (e.g. by including a probability or confidence associated with the range).
- c)** (3 points) When looking at the series of kicks with the helium-filled football, it appears that some of the kicks were uncharacteristically short (e.g., below 15 yards). These may be “misses” that should be considered as outliers. In order to try to give a statistical assessment of whether kicks below 15 yards could be considered as outliers, compute the probability of a single kick to be below 15 yards when assuming the distances obtained

with the helium-filled ball to be approximately normally distributed with the parameters estimated from the data. Does your computed probability aid in assessing whether these kicks should be considered as outliers? Why or why not? If you think the computed probability is not particularly helpful in itself, can you suggest a supplementary or alternative calculation that would provide useful (additional) information to decide about potential outliers. Explain (briefly) how you would use any such information obtained.

- d) (3 points) The question of main interest was whether the helium-filled football would travel further than the air-filled football. Give a quantitative assessment of this question by estimating the difference in the mean distances of kicks with the helium-filled and air-filled footballs. Supplement the estimate of this parameter with a 90% confidence interval. Draw conclusions from relevant statistical inference for this question.
- e) (3 points) Another way of comparing the performance of the two footballs is to determine, for each of the 39 kicks with both footballs, which of the kicks with the two footballs was the longer one. Use the data to estimate the probability that a kick with the helium-filled football will be longer than a (simultaneous) kick with an air-filled football. Compute a probability (a P -value for a statistical test) to assess whether the data provide any evidence that kicks with the helium-filled football are longer than simultaneous kicks with an air-filled football, and draw conclusions.

Minitab graphical display:



Minitab listings:

Descriptive Statistics: Air, Helium, Diff

Statistics

Variable	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum	Skewness
Air	39	25.923	4.687	15.000	23.000	26.000	29.000	35.000	-0.37
Helium	39	26.385	6.214	11.000	24.000	28.000	30.000	39.000	-0.88
Diff	39	0.46	6.87	-14.00	-2.00	1.00	4.00	17.00	-0.12

One-Sample T: Air, Helium, Diff

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean	90% CI for μ
Air	39	25.923	4.687	0.751	(24.658, 27.188)
Helium	39	26.385	6.214	0.995	(24.707, 28.062)
Diff	39	0.46	6.87	1.10	(-1.39, 2.32)

μ : mean of Air, Helium, Diff

Two-Sample T-Test and CI: Helium, Air

Method

μ_1 : mean of Helium

μ_2 : mean of Air

Difference: $\mu_1 - \mu_2$

Equal variances are not assumed for this analysis.

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Helium	39	26.38	6.21	1.0
Air	39	25.92	4.69	0.75

Estimation for Difference

Difference	90% CI for Difference
0.46	(-1.62, 2.54)

Stata listings:

```
. tabstat air helium diff, statistics( count mean sd min p25 p50 p75 max skewness )
```

stats	air	helium	diff
N	39	39	39
mean	25.92308	26.38462	.4615385
sd	4.686962	6.213802	6.866571
min	15	11	-14
p25	23	24	-2
p50	26	28	1
p75	29	30	4
max	35	39	17
skewness	-.351716	-.8427391	-.1181353

```
. ci mean air helium diff, level(90)
```

Variable	Obs	Mean	Std. Err.	[90% Conf. Interval]	
air	39	25.92308	.7505145	24.65774	27.18841
helium	39	26.38462	.9950047	24.70708	28.06215
diff	39	.4615385	1.099531	-1.392221	2.315298

```
. ttest helium==air, unequal unpaired level(90)
```

Two-sample t test with unequal variances

Variable	Obs	Mean	Std. Err.	Std. Dev.	[90% Conf. Interval]	
helium	39	26.38462	.9950047	6.213802	24.70708	28.06215
air	39	25.92308	.7505145	4.686962	24.65774	27.18841
combined	78	26.15385	.6196572	5.472664	25.12219	27.1855
diff		.4615385	1.246317		-1.615703	2.53878

... (output omitted)